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Principal motion ellipsoids: Gait variability index based on principal motion analysis

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Abstract— **We developed a gait variability index on the basis of principal motion analysis (PMA). PMA decomposes multivariate time-series data into independent bases. The resulting variability index evaluates how individual gaits vary along these bases. Hence, it allows us to analyze how and to what extent the gait motions of humans vary, unlike previous popular indices, which aimed to quantify the level of gait variability. We computed this variability index for lower limb joints and the velocity of the center of mass during walking. These indices exhibited higher correlation coefficients with MeanSD, which is a typical gait variability index. The meanings of the independent bases was successfully interpreted, suggesting the validity and usability of the new gait variability index.**

I. INTRODUCTION

There is a large variability in each individuals' walking, and this gait variability can be related to the risk of fall [1]. England et al. and Bruijn et al. discussed gait variability in terms of walking speed [2], [3]. Owings et al. focused on the variability of the step length during walking [4]. There are some studies that discussed gait variability within individuals not only during straight walking, but also during curving motion [5], [6]. Thus, many researchers have already studied gait variability. Indices to represent gait variability are invaluable in generalizing these research works and enhancing commercial applications.

MeanSD [7] and the maximum Lyapunov exponent [8] are the most popular indices to evaluate gait variability. MeanSD is the mean value of standard deviation of parameters such as joint angles. The maximum Lyapunov exponent quantifies[1] the average logarithmic rate of the divergence of a system after a perturbation, and it is often calculated by using the information from an accelerometer or internal measurement unit attached to the waist. If the values of these indices are large, the gait variability within individuals is large. However, given that these indices are represented by single scalar quantities, they are not suitable to discuss how walking motions vary, although they are certainly suitable for discussing the level of gait variability.

Human motions are sometimes analyzed by principal motion analysis (PMA) [9]. This method decomposes human motions into bases called principal motions without losing the information of interlocked multiple degrees of freedom of redundant systems. This method has mainly been applied to generate human and robot motions to reduce the dimensions of parameter spaces [9]–[13]. PMA has also been adapted to classify and identify individual gaits [14] and sit-to-stand motions [15].

This study aims to develop a new index for gait variability that is inherently represented by multivariate time series. For this purpose, we analyze the gait variability within individuals by applying PMA on walking data measured by a camera-based motion capture system. PMA allows us to analyze the variability that occurs during the walking phase. Therefore, our index represents both the level and quality of variability, i.e., represents walking motion varies how and at which phase within the gait cycle. We compute the variability indices using the joint angles or the velocities of the center of mass (CoM) during walking and verify the validity of the gait variability index. For this purpose, MeanSD and our index based on PMA are compared.

The experiment in this study was conducted with the approval of the Institutional Review Board of the School of Engineering, Nagoya University (#18–2).

II. EXPERIMENT OF WALKING MOTION

A. Experimental method and participants

A walking experiment was conducted on 12 adults (height: 172.5 ± 3.8 cm, weight: 63.6 ± 7.2 kg), having no neurological and musculoskeletal abnormalities. Written consent was obtained prior to the experiment. The participants were instructed to repeat straight walk, within a distance of approximately 5 m, with their faces turned to the front.

B. Motion data analysis

We extracted a motion–defined gait cycle from continuous walking by considering data from the left heel contact to the next left heel contact. The gait cycle was normalized to 0–100% to align the data length of each trial (Fig. 1). The measurement space was defined in the reference coordinate system, taking the horizontal direction as the x-axis (right for positive), the anterior posterior direction as the y-axis (anterior for positive), and the vertical direction as the z-axis (upper for positive) according to the right-handed system. The values of joint angles were positive in the direction of flexing and negative in the direction of extending from the basic standing posture. We analyzed hip flexion angles, knee flexion angles, and ankle dorsiflexion angles for the left and right legs. We defined the hip flexion angle as the angle between the vertical line passing through the hip joint and the straight line connecting the hip joint and the knee joint. The joint angles were calculated by fitting the motion of the

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[1]The maximum Lyapunov exponent is also used as a gait stability index.

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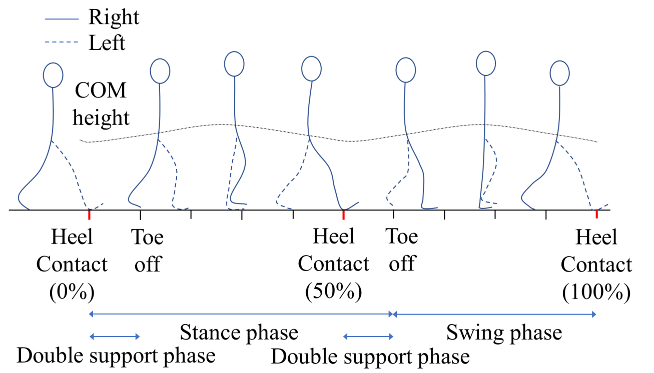

Fig. 1. Gait cycle normalized to 0–100%. Defined from the left heel contact (0%) to the next left heel contact (100%).

markers to the human model of SIMM (Mulsculographics Inc., U.S.) by using the least-squares method. We also analyzed the velocities of the center of mass (CoM) in the direction of the three axes. These velocities of the CoM were obtained by differentiating the position of the CoM of the whole body calculated using Zatsiorsky's method [16].

III. GAIT VARIABILITY INDICES

A. Gait variability indices based on principal motion analysis

1) Principal Motion Analysis (PMA): Park et al. [9] extended principal component analysis (PCA) to make it applicable to time-series data and achieved a representation of human motions with a reduced number of variables. PCA is a multivariate analysis method that identifies the combinations of correlated variables, which are called principal components. These components, which are calculated from the same data pool, are linearly independent of each other. PMA interlocks the variables along the temporal dimension as well as among multiple variables. The combination of interlocked variables in the temporal direction is called the principal motion. PMA is a linear analysis method, and it is possible to interpret the meanings of the obtained principal motions.

Suppose that a human motion at a certain instant is represented by p variables. For variable i $(i = 1, ..., p)$ at k-th trial $(k = 1, ..., k')$, the time-series data vector $\boldsymbol{\theta}_{ik}$, which consist of discretized u moments, is given as

$$\boldsymbol{\theta}_{ik} = (\theta_{ik1}, ..., \theta_{ikl}, ..., \theta_{iku})^{\mathrm{T}}. \tag{1}$$

Using this, we create an extended column vector $\boldsymbol{x}_k$ consisting of p variables as

$$\boldsymbol{x}_k = (\boldsymbol{\theta}_{1k}{}^{\mathrm{T}}, ..., \boldsymbol{\theta}_{ik}{}^{\mathrm{T}}, ..., \boldsymbol{\theta}_{pk}{}^{\mathrm{T}})^{\mathrm{T}}. \tag{2}$$

Note that $\boldsymbol{x}_k^{(s)}$ represents a time-series column vector of the experimental participant s $(s = 1, ..., s')$ at trial k. If we obtain k' sets of walking motion from each s' participants as motion samples, the time-series data of all the motions are represented by a matrix $\boldsymbol{X}$ $(\in \mathbb{R}^{s'k' \times pu})$ as follows:

$$\boldsymbol{X} = (\boldsymbol{x}_1^{(1)} - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}_k^{(s)} - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}_{k'}^{(s')} - \bar{\boldsymbol{x}})^{\mathrm{T}} \tag{3}$$

here, $\bar{\boldsymbol{x}}$ represents the mean value of the motions of all the participants.

From the eigenvector expansion of covariance matrix $\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}$, we obtain

$$\boldsymbol{X} \sim \boldsymbol{Y}\boldsymbol{V}^{\mathbf{T}}, \tag{4}$$

where $\boldsymbol{Y} = (\boldsymbol{y}_1^{(1)}, ..., \boldsymbol{y}_{k'}^{(s')})^{\mathrm{T}}$ is a score matrix that represents how much information of each principal motion is included by each observed motion and $\boldsymbol{V} = (\boldsymbol{v}_1, ..., \boldsymbol{v}_q, ..., \boldsymbol{v}_r)$ is a principal motion matrix composed of r eigenvectors corresponding to the r largest eigenvalues; $\boldsymbol{v}_q$ $(\in \mathbb{R}^{s'k' \times 1})$ is the q-th principal motion vector whose eigenvalue is the q-th largest. Any motion can be represented by a linear combination of the principal motion vectors weighted by their corresponding scores:

$$\boldsymbol{x}_k^{(s)} \sim \boldsymbol{V}\boldsymbol{y}_k^{(s)}. \tag{5}$$

2) Extraction of the repetition error within individuals: The variability of the walking motion is evaluated by considering the repetition errors within individuals. A matrix of repetition errors within individuals $\boldsymbol{B}$ $(\in \mathbb{R}^{s'k' \times pu})$ is computed as

$$\boldsymbol{B} = (\boldsymbol{x}_1^{(1)} - \bar{\boldsymbol{x}}^{(1)}, ..., \boldsymbol{x}_k^{(s)} - \bar{\boldsymbol{x}}^{(s)}, ..., \boldsymbol{x}_{k'}^{(s')} - \bar{\boldsymbol{x}}^{(s')})^{\mathrm{T}}, \tag{6}$$

where $\bar{\boldsymbol{x}}^{(s)}$ represents the mean of the motion of experimental participant s . A score matrix $\boldsymbol{Y}_b$ and a principal motion matrix $\boldsymbol{V}_b$ of this matrix $\boldsymbol{B}$ are obtained from the eigenvector expansion of covariance matrix $\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}$ as follows:

$$\boldsymbol{B} \sim \boldsymbol{Y}_b\boldsymbol{V}_b^{\mathrm{T}}. \tag{7}$$

We use the size of the error ellipsoid calculated individually from the distribution of the principal motion scores $\boldsymbol{Y}_b$ as a gait variability index. We let this error ellipsoid be the principal motion ellipsoid (PM ellipsoid). Suppose that $a_m^{(s)}$ represents the length of the major axis m $(m = 1, ..., m')$ of the PM ellipsoid of participant s with m' being the number of principal motions calculated from $\boldsymbol{B}$. Also, $a_m^{(s)}$ is the square root of the eigenvalue of the covariance matrix of the scores of participant s. We define the size of the PM ellipsoid $c^{(s)}$ by using the length of each axis as

$$c^{(s)} = \sum_{i=1}^{m'} a_m^{(s)}. \tag{8}$$

Fig. 2 shows the definition of the size of PM ellipsoid for participant s when using the first and second principal motions. In the latter example, we used three principal motions.

B. MeanSD

Many indices evaluating variability are based on the standard deviation of the analyzed variables [1]. In this study, we explore MeanSD as a standard index of gait variability that employs standard deviation. MeanSD is an mean value of standard deviation based on the gait cycle. It has been used so far for the joint angle of the lower limbs [7], [17], [18] and for velocity [3], [17], [19], [20] and acceleration [7],

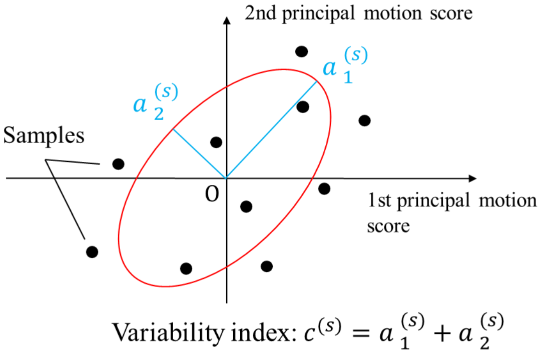

Fig. 2. Definition of the size of PM ellipsoid for participant s. An error ellipsoid (red) is calculated from the distributed samples (black dots).

[21] obtained from an accelerometer attached to the waist. For a participant, $\mathrm{SD}(u)$ represents the standard deviation at the moment u on a gait cycle of a variable. Then, MeanSD, which represents gait variability on the whole gait cycle of the variable, is defined as

$$\mathrm{MeanSD} = \langle \mathrm{SD}(u) \rangle, u \in \{0 - 100\% \, \text{gait cycle}\} \quad (9)$$

where $\langle \, \rangle$ represents the mean value of all the temporal points [7]. We computed MeanSD for joint angles and velocities of the CoM. We calculated each MeanSD of six angles for the joint angles and velocities of three directions for the velocity of the CoM, respectively, and we defined their mean value as MeanSD of the joint angle and the velocity of the CoM.

IV. Results

In this study, we analyzed the 1st–3rd principal motions. Fig. 3 shows the relationship between MeanSD and the size of the PM ellipsoid c computed from the six joint angles (top) and the three velocity components (x, y, and z) of the CoM (bottom). The correlation coefficient for MeanSD and c for the joint angles is 0.96 and the correlation coefficient for MeanSD and c for the velocity of the CoM is 0.95. These results indicate that MeanSD and the size of the PM ellipsoid are similar indices for the degree of gait variability.

V. Discussion

A. Interpreting principal motions for joint angles

We interpret how the gait variability within individuals exposes by interpreting each principal motion. Fig. 4 shows the time-series change of the loads of the 1st–3rd principal motion. Given that the load represents gait variability within individuals, if the load is 0, the variability is small, and if the load is positive or negative, the joint angle varies. If the load at a particular time instant is positive, the joint angle flexes more than the mean of the individual. The contribution ratio of each principal motion is as follows: for the 1st principal motion it is 29.3%, for the 2nd principal motion it is 11.1%, and for the 3rd principal motion it is 8.0%.

Top of Fig. 4, which represents the 1st principal motion, shows that the hip and knee joints tend to flex (positive),

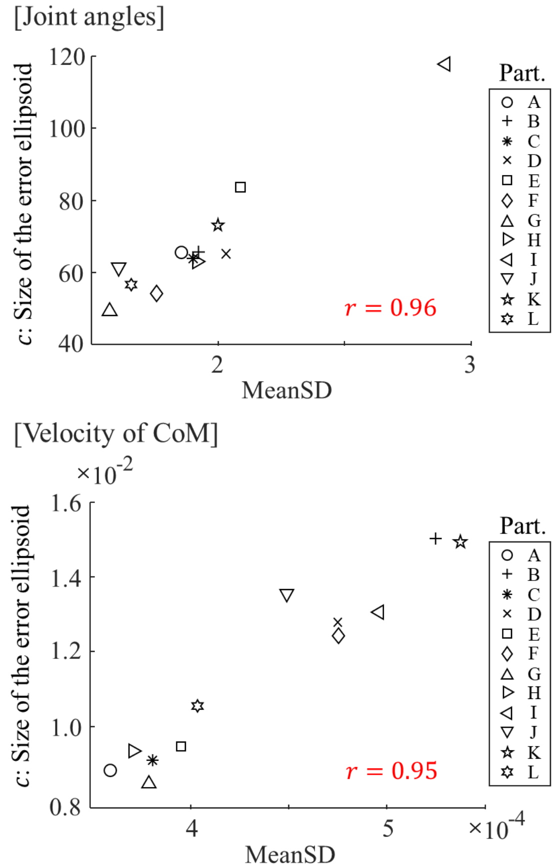

Fig. 3. The relationship between MeanSD and the size of the PM ellipsoid c computed from the six joint angles (top) and the three velocity components (x, y, and z) of the CoM (bottom).

and the ankle joints tend to plantarflex (negative) in the first half of the swing phase whereas the knee joints tend to extend (negative) in the second half of the swing phase. It can be considered that this motion represents the advance of the phase because this motion seems to be differentiated the average walking motion. Fig. 5 shows the knee flexion angles of gait samples with maximum and minimum scores of the 1st principal motion for one participant. Gait samples with maximum scores of the 1st principal motion shifted the gait phase earlier than minimum ones. Therefore, we can interpret that the 1st principal motion is the principal motion related to the speed of shifting the gait phase.

The walking motion with positive 1st principal motion score shifts to the free leg quickly. Conversely, the walking motion with negative score shifts to the free leg slowly. The variance of the gait sample along the 1st principal motion is greater than the variance along the other main motions, and the 1st principal motion indicates main variance of walking motions for many participants. Variations in the length of the standing and swing phases for this motion as described above account for a large proportion in gait variation.

Middle of Fig. 4, which represents the 2nd principal motion, shows that the hip and knee joint of the right leg tend to flex (positive), and the ankle joint of the right leg tends

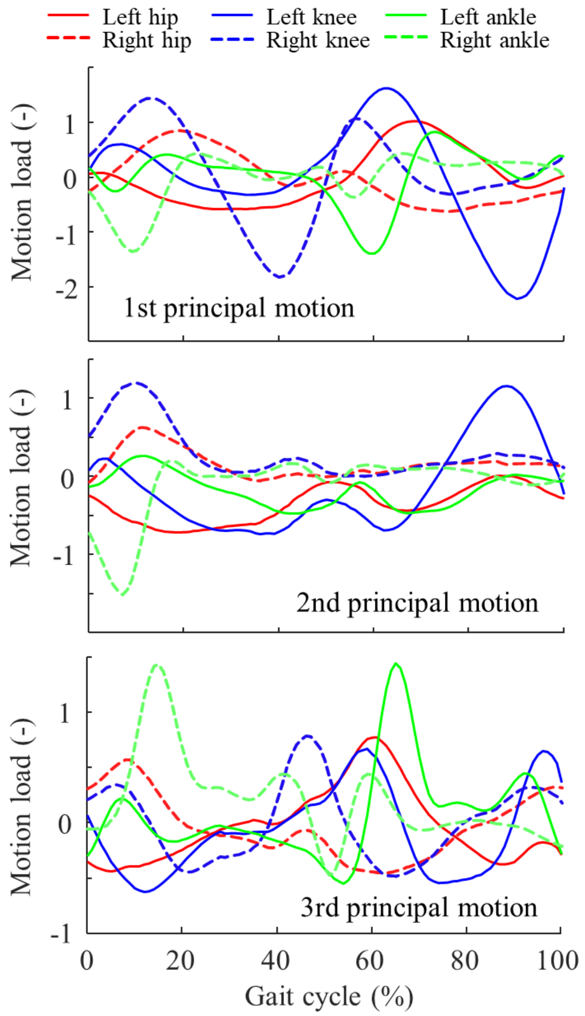

Fig. 4. Loads of each principal motion computed from joint angles. Top: 1st principal motion. Middle: 2nd principal motion. Bottom: 3rd principal motion.

to plantarflex (negative) in the first half of the gait phase whereas the knee joint of the left leg tends to flex (positive) in the second half of the gait phase. This motion means that the left leg flexes significantly more than the average motion in the latter half of the swing phase, and as compensation for it, the hip and knee joints flex and the ankle joint plantarflex at the moment of the right toe gets off. In short, this motion means that the left step length is smaller than the right step length. Therefore, the 2nd principal motion represents left-right asymmetry.

Bottom of Fig. 4, which represents the 3rd principal motion, shows that the ankle joints of both of legs tend to dorsiflex (positive), and the knee joints of both of legs tend to flex (positive) just before heel contact. This motion has long double stance phase and this is common for walking with a small step length. Therefore, the 3rd principal motion represents the step length.

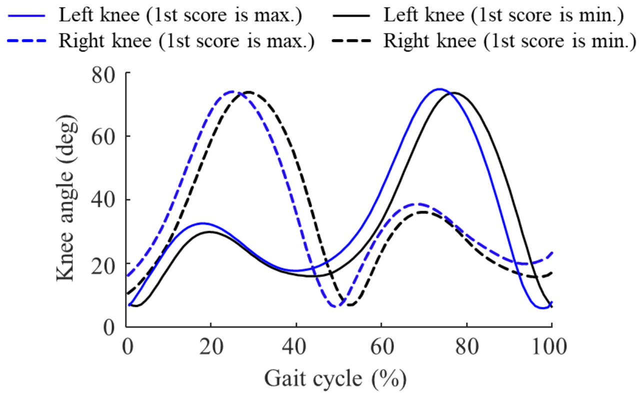

Fig. 5. Gait samples with maximum (blue line) and minimum (black line) scores of the 1st principal motion. Solid line: left knee angle. Dotted line: right knee angle.

B. *Interpreting principal motions for the velocity of the CoM*

Fig. 6 shows the time-series change of the loads of the 1st–3rd principal motion. As discussed above, the load represents gait variability within individuals. If the load at a particular time instant is positive, it indicates that the velocity of the CoM is larger than the mean of the individual. The contribution ratio of each principal motion is as follows: for the 1st principal motion it is 40.3%, for the 2nd principal motion it is 7.8%, and for the 3rd principal motion it is 5.9%.

Top of Fig. 6, which represents the 1st principal motion, shows that the velocity of the CoM in the anterior direction (y axis) is large (positive) throughout a gait cycle. Therefore, we can interpret that the 1st principal motion is related to the walking velocity towards anterior direction. The 1st principal motion represents the main variance of walking motions for many participants. Hence, for the velocity of the CoM, variations in the walking velocity towards the anterior direction account for a large proportion of gait variation.

Middle of Fig. 6, which represents the 2nd principal motion, shows that the velocity of the CoM in the horizontal (x axis) and anterior (y axis) directions is large (positive) in the left stance phase, whereas the velocity of the CoM in the horizontal (x axis) and anterior (y axis) directions is small (negative) in the right stance phase. The right step length is larger than the left step length and the CoM is close to the swing leg. Fig. 7 shows the angles of gait samples with maximum and minimum scores of the 2nd principal motion of one participant. In the walking sample with the large 2nd principal motion score, the left knee angle is relatively small compared with the right. Therefore, the second principal motion represents left-right asymmetry.

Given that the 3rd principal motion is the motion that differentiated the average walking motion, as the bottom of Fig. 6 shows, it can be considered that this motion represents the advance of the phase. Therefore, we can interpret that the 3rd principal motion is related to the speed of gait phase shifting.

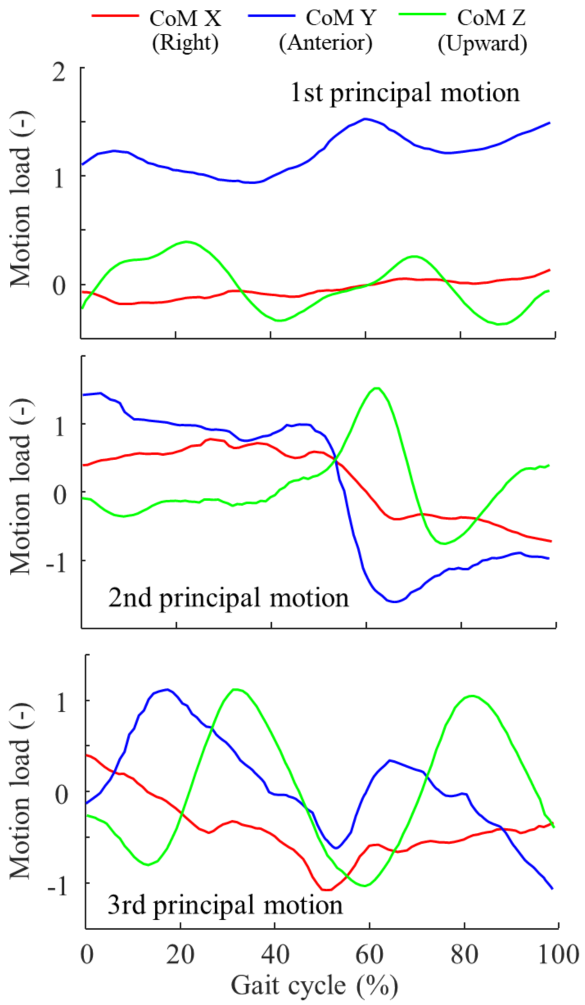

Fig. 6. Loads of each principal motion computed from the velocity of the CoM. Top: 1st principal motion. Middle: 2nd principal motion. Bottom: 3rd principal motion. CoM X, Y, and Z means the velocity of the CoM in x, y, and z directions, respectively.

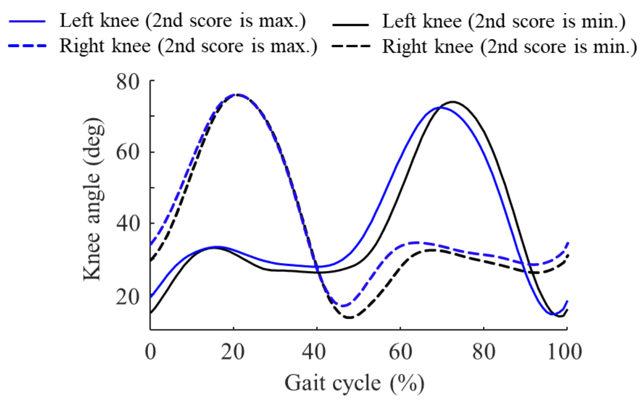

Fig. 7. Gait samples with maximum (blue line) and minimum (black line) scores of the 2nd principal motion. Solid line: left knee angle. Dotted line: right knee angle.

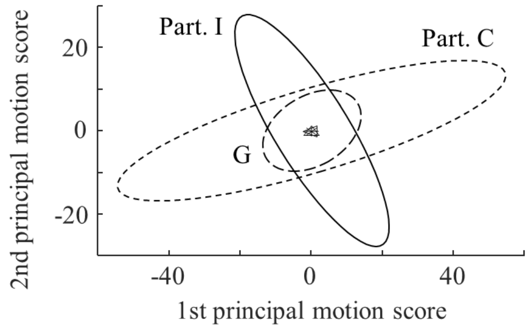

Fig. 8. Error ellipsoids of participants C, G, and I obtained from principal motion scores. The size and the direction of major axes of the error ellipsoids differ among participants.

C. Relationship of PM ellipsoid and gait variability

We analyzed the variations in walking within individuals by using PM ellipsoid as an index of gait variability. The 1st principal motion of joint angles and the 3rd motion of velocities of the CoM represent the speed of gait phase shifting, whereas the 2nd principal motion of joint angles and velocities of the CoM represent left-right asymmetry.

In Fig. 8, the PM ellipsoids for joint angles obtained from the experiment are shown on the 1st and 2nd principal motion plane. Participant G was the participant whose PM ellipsoid was smallest whereas participant C and I exhibited the largest PM ellipsoids. The direction of the major axis of the PM ellipsoid of participant C was mostly along the 1st principal motion whereas that of participant I was mostly along the 2nd principal motion. This indicates that the gait variability of participant C is mainly related to the speed of gait phase shifting and the gait variability of participant I is mainly related to left-right asymmetry. The size and direction of major axes of the PM ellipsoids differed among participants. Thus, each principal motion represents how walking motions vary within individuals. MeanSD and Lyapunov exponent are scalar values to express the size of walking variation. PM ellipsoids also express how gait motions vary. From this viewpoint, PM ellipsoids can be used as a general index to obtain walking variations.

VI. Conclusion

We proposed the size of the PM ellipsoid as a new gait variability index. We compared MeanSD, which is a representative gait variability evaluation index that has been commonly used in the literature, with the size of the PM ellipsoid obtained from the principal motion score. As a result, there was a high correlation between the two indices when we considered both joint angles of lower limbs and velocities of the CoM. The variability index based on PMA is a useful index because after understanding how walking motions vary, we can interpret the meanings of the principal motions.

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